Introduction to Engineering Materials
ENGR2000
Chapter 6: Mechanical Properties of Metals
Dr. Coates
6.2 Concepts of Stress and Strain

**tension**

**compression**

**shear**

**torsion**
Tension Tests

- The specimen is deformed to fracture with a gradually increasing tensile load...
Fracture & Failure

- **Fracture** occurs when a structural component separates into two or more pieces.
  - fracture represents failure of the component
- **Failure** is the inability of a component to perform its desired function.
  - failure may occur prior to fracture
  - e.g. a car axle that bends when you drive over a pothole
Engineering Stress & Engineering Strain

Engineering stress:

\[ \sigma = \frac{F}{A_0} \]

Engineering strain:

\[ \varepsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} \]

*F*-instantaneous load applied \( \perp \) to specimen cross section

\( A_0 \)-original cross section area before any load is applied

\( l_i \)-instantaneous length \( l_0 \)-original length
If you double the cross sectional area, would your load needed to produce a given elongation be the same?

Why not just use force $F$ and deformation $\Delta l$ to study mechanical behavior?
Compression Tests

• Similar to the tensile tests
• Force is compressive
• Specimen contracts along direction of the stress
Engineering Stress & Engineering Strain

Engineering stress:

\[ \sigma = -\frac{F}{A_0} \]

Engineering strain:

\[ \varepsilon = \frac{\Delta l}{l_0} = \frac{l-l_0}{l_0} < 0 \]
Shear and Torsional Tests

Shear stress:
\[
\tau = \frac{F}{A_0}
\]

Shear strain (by definition):
\[
\gamma = \tan \theta \cong \theta \quad (\text{for small } \theta)
\]

\(F\)-instantaneous load applied // to specimen cross section
Shear and Torsional Tests

Shear stress due to applied torque:
\[ \tau = f(T) \]

Shear strain due to applied torque:
\[ \gamma = f(\phi) \]
Stress Transformation

A force balance in the $y'$ direction yields:

$$(\sigma_y A) \cos \theta = \sigma_{y'} \left( \frac{A}{\cos \theta} \right)$$

$$\sigma_{y'} = \sigma_y \cos^2 \theta$$

A force balance in the tangential direction yields:

$$\tau_{x'y'} = \sigma_y \sin \theta \cos \theta$$
Deformation

- All materials undergo changes in dimensions in response to mechanical forces. This phenomenon is called deformation.
  - If the material reverts back to its original size and shape upon removal of the load, the deformation is said to be elastic deformation.
  - If application and removal of the load results in a permanent change in size or shape, the deformation is said to be plastic deformation.
Elastic Deformation
6.3 Stress-Strain Behavior

Hooke's law:

\[ \sigma = E \varepsilon \]

Modulus of elasticity:

\[ E = \frac{\sigma}{\varepsilon} \]

Shear modulus:

\[ G = \frac{\tau}{\gamma} \]

ELASTIC DEFORMATION!
Modulus of Elasticity

- Young’s modulus
- Elastic modulus
- Stiffness
- Material’s resistance to elastic deformation
  - greater the elastic modulus, the stiffer the material or smaller the elastic strain that results from the application of a given stress.
Tangent or Secant Modulus

- Materials with non-linear elastic behavior
  - Gray cast iron,
  - concrete,
  - many polymers
Elastic modulus of metals, ceramics & polymers

- **Ceramics**
  - high elastic modulus
  - diamond, graphite (covalent bonds)

- **Metals**
  - high elastic modulus

- **Polymers**
  - low elastic modulus
  - weak secondary bonds between chains
Elastic Modulus on an Atomic Scale

• Elastic strain
  – small changes in inter-atomic spacing
  – stretching of inter-atomic bonds

• Elastic modulus
  – resistance to separation of adjacent atoms
  – inter-atomic bonding forces
Review

2.5 Bonding Forces and Energies

The net force:

\[ F_N = F_A + F_R \]

At equilibrium:

\[ F_N = F_A + F_R = 0 \]

- the centers of the two atoms are \( r_0 \) apart.

\( r_0 \) is known as the bond length.
Figure 2.8 (a) The dependence of repulsive, attractive, and net forces on interatomic separation for two isolated atoms. (b) The dependence of repulsive, attractive, and net potential energies on interatomic separation for two isolated atoms.
Elastic Modulus on an Atomic Scale

How might $E$ be related to $\left(\frac{dF}{dr}\right)_{r_0}$? Why?

**Figure 6.7** Force versus interatomic separation for weakly and strongly bonded atoms. The magnitude of the modulus of elasticity is proportional to the slope of each curve at the equilibrium interatomic separation $r_0$. 
Effect of Temperature on the Elastic Modulus

**Figure 6.8** Plot of modulus of elasticity versus temperature for tungsten, steel, and aluminum. (Adapted from K. M. Ralls, T. H. Courtney, and J. Wulff, *Introduction to Materials Science and Engineering*. Copyright © 1976 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.)
<table>
<thead>
<tr>
<th>Metal Alloy</th>
<th>Modulus of Elasticity</th>
<th>Shear Modulus</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GPa</td>
<td>$10^6$ psi</td>
<td>GPa</td>
</tr>
<tr>
<td>Aluminum</td>
<td>69</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>Brass</td>
<td>97</td>
<td>14</td>
<td>37</td>
</tr>
<tr>
<td>Copper</td>
<td>110</td>
<td>16</td>
<td>46</td>
</tr>
<tr>
<td>Magnesium</td>
<td>45</td>
<td>6.5</td>
<td>17</td>
</tr>
<tr>
<td>Nickel</td>
<td>207</td>
<td>30</td>
<td>76</td>
</tr>
<tr>
<td>Steel</td>
<td>207</td>
<td>30</td>
<td>83</td>
</tr>
<tr>
<td>Titanium</td>
<td>107</td>
<td>15.5</td>
<td>45</td>
</tr>
<tr>
<td>Tungsten</td>
<td>407</td>
<td>59</td>
<td>160</td>
</tr>
</tbody>
</table>

Table 6.1  Room-Temperature Elastic and Shear Moduli, and Poisson’s Ratio for Various Metal Alloys
Example 6.1

• A piece of Cu originally 305 mm long is pulled in tension with a stress of 276 Mpa. If the deformation is entirely elastic, what will be the resultant elongation?
Example 6.1

Given:

\( l_0 = 305 \, mm \)
\( \sigma = 276 \, MPa \)

Using Table 6.1:
\( E_{cu} = 110 \, GPa \)
\( \Delta l = ? \)

Using Hooke's law:

\[
\sigma = E\varepsilon = E \frac{\Delta l}{l_0}
\]

\( \Rightarrow \Delta l = \sigma \frac{l_0}{E} = 0.77 \, mm \)
6.4 Anelasticity
- time dependent elastic behavior

- Time dependent elastic strain component.
- Elastic deformation continues after the stress application.
- Upon load release, finite time is required for complete recovery.
Anelasticity of metals, ceramics & polymers

- Polymers
  - Visco-elastic behavior
- Metals

Increasing anelasticity
6.5 Elastic Properties of Materials

**Figure 6.9** Axial (z) elongation (positive strain) and lateral (x and y) contractions (negative strains) in response to an imposed tensile stress. Solid lines represent dimensions after stress application; dashed lines, before.

\[ \frac{\varepsilon_z}{2} = \frac{\Delta l_z/2}{l_{0z}} \]

\[ -\frac{\varepsilon_x}{2} = \frac{\Delta l_x/2}{l_{0x}} \]
Poisson’s Ratio

Poisson's ratio (a material constant):

\[ \nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z} \]

\[ \nu > 0 \]

\[ \nu_{\text{theoretical}} = \frac{1}{4} = 0.25 \]

\[ \nu_{\text{max}} = 0.50 \text{ (no net volume change)} \]

For most metals:

\[ 0.25 \leq \nu \leq 0.35 \]
Elastic Properties of Materials

Elastic constants (material properties):

\( E, \ G, \ \nu \)

For isotropic materials (same properties in all directions):

\[
G = \frac{E}{2(1+\nu)}
\]
Example 6.2

• A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm. Determine the magnitude of the load required to produce a $2.5 \times 10^{-3}$ mm change in diameter if the deformation is entirely elastic.
Given:
\(d_0 = 10 \text{mm}\)
\(\Delta d = 2.5 \times 10^{-3} \text{mm}\)

Using Table 6.1:
\(E_{\text{brass}} = 97 \text{GPa}\)
\(\nu_{\text{brass}} = 0.34\)
\(F = ?\)
Compute the strain:
\[ \varepsilon_x = \frac{\Delta d}{d_0} = -2.5 \times 10^{-4} \]

Using the Poisson's ratio:
\[ \varepsilon_z = -\frac{\varepsilon_x}{\nu} = 7.35 \times 10^{-4} \]

Using Hooke's law:
\[ \sigma = \varepsilon_z E = 71.3 \text{ MPa} \]

Compute the force:
\[ F = \sigma \left( \pi \frac{d_0^2}{4} \right) = 5600 \text{ N} \]
Plastic Deformation

- Elastic deformation
  - Up to strains of 0.005

- Plastic deformation
  - Hooke’s law is no longer valid
  - Non-recoverable or permanent deformation
  - Phenomenon of yielding
6.6 Tensile Properties

\[ \sigma_{ys} - \text{yield strength (stress corresponding to the elastic limit)} \]

\[ \varepsilon_{yp} - \text{yield point strain} \]

\[ \sigma_{uts} - \text{ultimate tensile strength (maximum engineering stress)} \]

\[ \varepsilon_u - \text{uniform strain} \]

\[ \sigma_f - \text{fracture strength (stress at fracture)} \]

\[ \varepsilon_f - \text{strain at fracture} \]
Yielding and Yield Strength

- Gradual elastic-plastic transition
- Proportional limit (P)
  - Initial departure from linearity
  - Point of yielding
- Yield strength
  - Determined using a 0.002 strain offset method
Yielding and Yield Strength

- Steels & other materials
- Yield point phenomenon
- Yield strength
  - Average stress associated with the lower yield point

![Diagram](image.png)

- Stress
- Strain
- Upper yield point
- Lower yield point
- \( \sigma_y \)
Elastic deformation, plastic deformation, necking & fracture

**Figure 6.11**
Typical engineering stress–strain behavior to fracture, point $F$. The tensile strength $TS$ is indicated at point $M$. The circular insets represent the geometry of the deformed specimen at various points along the curve.
Example 6.3

- From the tensile stress-strain behavior for the brass specimen shown, determine the following:
  - The modulus of elasticity
  - The yield strength at a strain offset of 0.002
  - The maximum load that can be sustained by a cylindrical specimen having an original diameter of 12.8 mm
  - The change in length of a specimen originally 250 mm long that is subjected to a tensile stress of 345 Mpa.
• http://www.youtube.com/watch?v=67fSwIjYJ-E
Elastic modulus:

\[ E = \frac{\Delta \sigma}{\Delta \varepsilon} \]

\[ = \frac{150 - 0}{0.0016 - 0} \]

\[ = 93.8 \text{ GPa} \]

Yield strength:

\[ \sigma_{ys} = 250 \text{ MPa} \]
Given:

\( d_0 = 12.8 \text{ mm} \)

\( F_{\text{max}} = ? \)

Using the tensile strength:

\( \sigma_{\text{uts}} = 450 \text{ MPa} \)

\[ F_{\text{max}} = \sigma_{\text{uts}} \left( \frac{\pi d_0^2}{4} \right) = 57,900 \text{ N} \]
Given:

\( l_0 = 250 \text{ mm} \)

\( \sigma = 345 \text{ MPa} \)

\( \Delta l = ? \)

Determine the strain (point A):

\( \varepsilon \approx 0.06 \)

Compute the elongation:

\( \Delta l = \varepsilon l_0 = 15 \text{ mm} \)
Ductility

• Measure of the degree of plastic deformation that has been sustained at fracture.
• Brittle materials
  – Little or no plastic deformation prior to fracture
• Ductile materials
  – Significant plastic deformation prior to fracture
Ductility

Percent elongation:

\[ \% EL = \frac{l_f - l_0}{l_0} \times 100 = \varepsilon_f \times 100 \]

where \( l_0 \) is the original length
and \( l_f \) is the length at fracture.

Percent reduction in area:

\[ \% RA = \frac{A_0 - A_f}{A_0} \times 100 \]

where \( A_0 \) is the original cross-sectional area
and \( A_f \) is the final cross-sectional area of the necked region.
Ductility

• Why do we need to know the ductility of materials?
  – The degree to which a structure will deform plastically prior to fracture (design & safety measures)
  – The degree of allowable deformation during fabrication
Table 6.2  Typical Mechanical Properties of Several Metals and Alloys in an Annealed State

<table>
<thead>
<tr>
<th>Metal Alloy</th>
<th>Yield Strength</th>
<th>Tensile Strength</th>
<th>Ductility: %EL [in 50 mm (2 in.)]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPa (ksi)</td>
<td>MPa (ksi)</td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>35 (5)</td>
<td>90 (13)</td>
<td>40</td>
</tr>
<tr>
<td>Copper</td>
<td>69 (10)</td>
<td>200 (29)</td>
<td>45</td>
</tr>
<tr>
<td>Brass (70Cu–30Zn)</td>
<td>75 (11)</td>
<td>300 (44)</td>
<td>68</td>
</tr>
<tr>
<td>Iron</td>
<td>130 (19)</td>
<td>262 (38)</td>
<td>45</td>
</tr>
<tr>
<td>Nickel</td>
<td>138 (20)</td>
<td>480 (70)</td>
<td>40</td>
</tr>
<tr>
<td>Steel (1020)</td>
<td>180 (26)</td>
<td>380 (55)</td>
<td>25</td>
</tr>
<tr>
<td>Titanium</td>
<td>450 (65)</td>
<td>520 (75)</td>
<td>25</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>565 (82)</td>
<td>655 (95)</td>
<td>35</td>
</tr>
</tbody>
</table>
Mechanical properties of metals

• Depend on
  – prior deformation,
  – presence of impurities
  – heat treatments
Engineering stress-strain behavior for Fe at different temperatures
Resilience

• Capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered.

• Modulus of resilience
  – Strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding.
Resilience

Modulus of resilience:

\[ U_r = \int_{0}^{\varepsilon_y} \sigma d\varepsilon \]

Assuming a linear elastic region:

\[ U_r = \frac{1}{2} \sigma_y \varepsilon_y \]

\[ = \frac{1}{2} \sigma_y \frac{\sigma_y}{E} = \frac{\sigma_y^2}{2E} \]
Resilience

• Materials used in spring applications
  – High resilience
  – High yield strength
  – Low elastic modulus
Toughness

- Measure of the ability of a material to absorb energy up to fracture.
- Area under the stress-strain curve up to the point of fracture.
Toughness

Toughness:

\[ U = \int_{0}^{\varepsilon_f} \sigma d\varepsilon \]
Toughness

- Ductile materials
- Brittle materials

![Diagram showing stress-strain curve with increasing toughness](image)
Example

- Which material has the highest elastic modulus?
- Which material has the highest ductility?
- Which material has the highest toughness?
- Which material does not exhibit any significant plastic deformation prior to fracture?
6.7 True Stress and Strain

- Decline in stress necessary to continue deformation past the maximum point M.
- Is the material getting weaker?
6.7 True Stress and Strain

- Short-coming in the engineering stress-strain curve
  - Decreasing cross-sectional area not considered
6.7 True Stress and Strain

True stress:

\[ \sigma_t = \frac{F}{A_i} \]

where \( A_i \) is the instantaneous area.

True strain:

\[ \varepsilon_t = \int_{l_0}^{l} \frac{dl}{l} = \ln \left( \frac{l}{l_0} \right) \]
6.7 True Stress and Strain

Assuming no volume change:

\[ A_i l_i = A_0 l_0 \]

Prior to necking:

\[ \varepsilon_t = \ln(1 + \varepsilon) \]

\[ \sigma_t = \sigma (1 + \varepsilon) \]
6.7 True Stress and Strain

• Necking introduces a complex stress state…

**Figure 6.16** A comparison of typical tensile engineering stress–strain and true stress–strain behaviors. Necking begins at point $M$ on the engineering curve, which corresponds to $M'$ on the true curve. The “corrected” true stress–strain curve takes into account the complex stress state within the neck region.
Strain-hardening exponent

From the onset of plastic deformation to the point of necking:

\[ \sigma_t = K \varepsilon_t^n \]

where \( n \) is the strain-hardening exponent and \( K \) is the strength coefficient.
<table>
<thead>
<tr>
<th>Material</th>
<th>$n$</th>
<th>$K$ (MPa)</th>
<th>$K$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-carbon steel (annealed)</td>
<td>0.21</td>
<td>600</td>
<td>87,000</td>
</tr>
<tr>
<td>4340 steel alloy (tempered @ 315°C)</td>
<td>0.12</td>
<td>2650</td>
<td>385,000</td>
</tr>
<tr>
<td>304 stainless steel (annealed)</td>
<td>0.44</td>
<td>1400</td>
<td>205,000</td>
</tr>
<tr>
<td>Copper (annealed)</td>
<td>0.44</td>
<td>530</td>
<td>76,500</td>
</tr>
<tr>
<td>Naval brass (annealed)</td>
<td>0.21</td>
<td>585</td>
<td>85,000</td>
</tr>
<tr>
<td>2024 aluminum alloy (heat treated—T3)</td>
<td>0.17</td>
<td>780</td>
<td>113,000</td>
</tr>
<tr>
<td>AZ-31B magnesium alloy (annealed)</td>
<td>0.16</td>
<td>450</td>
<td>66,000</td>
</tr>
</tbody>
</table>
Example 6.4

• A cylindrical specimen of steel having an original diameter of 12.8 mm is tensile tested to fracture and found to have an engineering fracture stress of 460 Mpa. If its cross-sectional diameter at fracture is 10.7 mm, determine
  – The ductility in terms of percent reduction in area
  – The true stress at fracture
Given:

\( d_0 = 12.8 \text{ mm} \)
\( \sigma_f = 460 \text{ MPa} \)
\( d_f = 10.7 \text{ mm} \)
\( \% \text{RA} = ? \)
\( \sigma_{tf} = ? \)

Ductility:

\( \% \text{RA} = \frac{A_0 - A_f}{A_0} \times 100 \)
\[= \frac{d_0^2 - d_f^2}{d_0^2} \times 100 = 30\% \]
Engineering stress at fracture:

$$\sigma_f = 460 \text{ MPa} = \frac{F}{A_0}$$

The applied load:

$$F = \sigma_f A_0 = 59,200 \text{ N}$$

True stress at fracture:

$$\sigma_{tf} = \frac{F}{A_f} = 660 \text{ MPa}$$
Example 6.5

• Compute the strain-hardening exponent $n$ for an alloy in which a true stress of 415 Mpa produces a true strain of 0.10. Assume a value of 1035 Mpa for $K$. 
Example 6.5

Given:

\[ \sigma_t = 415 \text{ MPa} \]
\[ \varepsilon_t = 0.10 \]
\[ K = 1035 \text{ MPa} \]
\[ n = ? \]

Strain-hardening exponent:

\[ \sigma_t = K \varepsilon_t^n \]
\[ \Rightarrow n = 0.40 \]
6.10 Hardness

- Measure of a materials resistance to localized plastic deformation (small dent or scratch)
Rockwell Hardness Tests

• A hardness number is determined
  – Difference in depth of penetration from an initial minor load followed by a major load

• Combinations of indenters & loads are used
# Hardness Testing Techniques

## Table 6.4 Hardness-Testing Techniques

<table>
<thead>
<tr>
<th>Test</th>
<th>Indenter</th>
<th>Shape of Indentation</th>
<th>Load</th>
<th>Formula for Hardness Number&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brinell</td>
<td>10-mm sphere of steel or tungsten carbide</td>
<td>![Shape Diagram]</td>
<td>$P$</td>
<td>$\text{HB} = \frac{2P}{\pi D[(D - \sqrt{D^2 - d^2})]}$</td>
</tr>
<tr>
<td>Vickers microhardness</td>
<td>Diamond pyramid</td>
<td>![Shape Diagram]</td>
<td>$P$</td>
<td>$\text{HV} = \frac{1.854P}{d_1^2}$</td>
</tr>
<tr>
<td>Knoop microhardness</td>
<td>Diamond pyramid</td>
<td>![Shape Diagram]</td>
<td>$P$</td>
<td>$\text{HK} = \frac{14.2P}{l^2}$</td>
</tr>
<tr>
<td>Rockwell and</td>
<td>Diamond cone</td>
<td>![Shape Diagram]</td>
<td>$60 \text{ kg}$</td>
<td>Rockwell</td>
</tr>
<tr>
<td>Superficial</td>
<td>$\frac{1}{10}$ in. diameter steel spheres</td>
<td>![Shape Diagram]</td>
<td>$100 \text{ kg}$</td>
<td></td>
</tr>
<tr>
<td>Rockwell</td>
<td></td>
<td></td>
<td>$150 \text{ kg}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$15 \text{ kg}$</td>
<td>Superficial Rockwell</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$30 \text{ kg}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$45 \text{ kg}$</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> For the hardness formulas given, $P$ (the applied load) is in kg, while $D$, $d$, $d_1$, and $l$ are all in mm.

### Table 6.5a  Rockwell Hardness Scales

<table>
<thead>
<tr>
<th>Scale Symbol</th>
<th>Indenter</th>
<th>Major Load (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Diamond</td>
<td>60</td>
</tr>
<tr>
<td>B</td>
<td>$\frac{1}{16}$-in. ball</td>
<td>100</td>
</tr>
<tr>
<td>C</td>
<td>Diamond</td>
<td>150</td>
</tr>
<tr>
<td>D</td>
<td>Diamond</td>
<td>100</td>
</tr>
<tr>
<td>E</td>
<td>$\frac{1}{8}$-in. ball</td>
<td>100</td>
</tr>
<tr>
<td>F</td>
<td>$\frac{1}{16}$-in. ball</td>
<td>60</td>
</tr>
<tr>
<td>G</td>
<td>$\frac{1}{16}$-in. ball</td>
<td>150</td>
</tr>
<tr>
<td>H</td>
<td>$\frac{1}{8}$-in. ball</td>
<td>60</td>
</tr>
<tr>
<td>K</td>
<td>$\frac{1}{8}$-in. ball</td>
<td>150</td>
</tr>
</tbody>
</table>

Minor load = 10 kg
- used for thick specimens
### Table 6.5b

Superficial Rockwell Hardness Scales

<table>
<thead>
<tr>
<th>Scale Symbol</th>
<th>Indenter</th>
<th>Major Load (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15N</td>
<td>Diamond</td>
<td>15</td>
</tr>
<tr>
<td>30N</td>
<td>Diamond</td>
<td>30</td>
</tr>
<tr>
<td>45N</td>
<td>Diamond</td>
<td>45</td>
</tr>
<tr>
<td>15T</td>
<td>1/16-in. ball</td>
<td>15</td>
</tr>
<tr>
<td>30T</td>
<td>1/16-in. ball</td>
<td>30</td>
</tr>
<tr>
<td>45T</td>
<td>1/16-in. ball</td>
<td>45</td>
</tr>
<tr>
<td>15W</td>
<td>1/8-in. ball</td>
<td>15</td>
</tr>
<tr>
<td>30W</td>
<td>1/8-in. ball</td>
<td>30</td>
</tr>
<tr>
<td>45W</td>
<td>1/8-in. ball</td>
<td>45</td>
</tr>
</tbody>
</table>

**Minor load = 3 kg**
- thin specimens
Correlations between hardness & tensile strength

- Both are measures of resistance to plastic deformation
Property Variability & Design/Safety Factors

• Reading assignment
  – Section 6.11 on variability of material properties
  – Example 6.6
6.12 Design/Safety Factors

Design stress:

\[ \sigma_d = N' \sigma_c \]

\( \sigma_c \) is the calculated stress (on the basis of the estimated max load)

\( N' > 1 \) is the design factor.

Material selection criteria:

\[ \sigma_{ys} \geq \sigma_d \]
6.12 Design/Safety Factors

Safe stress or working stress:

$$\sigma_w = \frac{\sigma_{ys}}{N}$$

\(\sigma_{ys}\) is the yield strength of the material and \(N > 1\) is the factor of safety.

Factor of safety:

$$1.2 \leq N \leq 4.0$$
Design Example 6.1

- A tensile-testing apparatus is to be constructed that must withstand a maximum load of 220 kN. The design calls for two cylindrical support posts, each of which is to support half of the max load. Furthermore, plain carbon steel ground and polished shafting rounds are to be used; the minimum yield and tensile strengths of this alloy are 310 Mpa and 565 Mpa respectively. Specify a suitable diameter for these support posts.
Given:

\[ F_{\text{max}} = \frac{1}{2} (220 \text{kN}) = 110 \text{kN} \]

\[ \sigma_{\text{ys}} = 310 \text{MPa} \]

\[ \sigma_{\text{uts}} = 565 \text{MPa} \]

\[ d = ? \]

Assume a factor of safety:

\[ N = 5 \]

Working stress:

\[ \sigma_w = \frac{\sigma_{\text{ys}}}{N} = 62 \text{MPa} \]

Minimum diameter required:

\[ \sigma_w = \frac{F_{\text{max}}}{\pi d_0^2 / 4} \]

\[ \Rightarrow d_0 = 47.5 \text{mm} \]