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In preceding chapters, it was assumed that surfaces in contact were either *frictionless* (surfaces could move freely with respect to each other) or *rough* (tangential forces prevent relative motion between surfaces).

Actually, no perfectly frictionless surface exists. For two surfaces in contact, tangential forces, called *friction forces*, will develop if one attempts to move one relative to the other.

However, the friction forces are limited in magnitude and will not prevent motion if sufficiently large forces are applied.

The distinction between frictionless and rough is, therefore, a matter of degree.

There are two types of friction: *dry* or *Coulomb friction* and *fluid friction*. Fluid friction applies to lubricated mechanisms. The present discussion is limited to dry friction between nonlubricated surfaces.
The Laws of Dry Friction. Coefficients of Friction

- Block of weight $W$ placed on horizontal surface. Forces acting on block are its weight and reaction of surface $N$.

- Small horizontal force $P$ applied to block. For block to remain stationary, in equilibrium, a horizontal component $F$ of the surface reaction is required. $F$ is a **static-friction force**.

- As $P$ increases, the static-friction force $F$ increases as well until it reaches a maximum value $F_m$.

\[
F_m = \mu_s N
\]

- Further increase in $P$ causes the block to begin to move as $F$ drops to a smaller **kinetic-friction force** $F_k$.

\[
F_k = \mu_k N
\]
The Laws of Dry Friction. Coefficients of Friction

- Maximum static-friction force:
  \[ F_m = \mu_s N \]

- Kinetic-friction force:
  \[ F_k = \mu_k N \]
  \[ \mu_k \approx 0.75 \mu_s \]

- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area
• Four situations can occur when a rigid body is in contact with a horizontal surface:

- No friction, \((P_x = 0)\)
- No motion, \((P_x < F_m)\)
- Motion impending, \((P_x = F_m)\)
- Motion, \((P_x > F_m)\)
Angles of Friction

- It is sometimes convenient to replace normal force $N$ and friction force $F$ by their resultant $R$:

$$\tan \phi_s = \frac{F \mu_s}{N}$$
$$\tan \phi_s = \mu_s$$

$$\tan \phi_k = \frac{F \mu_k}{N}$$
$$\tan \phi_k = \mu_k$$
Angles of Friction

- Consider block of weight $W$ resting on board with variable inclination angle $\theta$.

- No friction
- No motion
- Motion impending
- Motion
Problems Involving Dry Friction

- All applied forces known
- Coefficient of static friction is known
- Determine whether body will remain at rest or slide

- All applied forces known
- Motion is impending
- Determine value of coefficient of static friction.

- Coefficient of static friction is known
- Motion is impending
- Determine magnitude or direction of one of the applied forces
A 100 lb force acts as shown on a 300 lb block placed on an inclined plane. The coefficients of friction between the block and plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the block is in equilibrium and find the value of the friction force.

**SOLUTION:**
- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.
- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.
- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.
Sample Problem 8.1

SOLUTION:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.

\[ \sum F_x = 0 : \quad 100 \text{ lb} - \frac{3}{5}(300 \text{ lb}) - F = 0 \]

\[ F = -80 \text{ lb} \]

\[ \sum F_y = 0 : \quad N - \frac{4}{5}(300 \text{ lb}) = 0 \]

\[ N = 240 \text{ lb} \]

- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.

\[ F_m = \mu_s N \]

\[ F_m = 0.25(240 \text{ lb}) = 60 \text{ lb} \]

- The block will slide down the plane.
Sample Problem 8.1

- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

\[ F_{actual} = F_k = \mu_k \cdot N \]

\[ = 0.20(240 \text{ lb}) \]

\[ F_{actual} = 48 \text{ lb} \]
Wedges

- Wedges - simple machines used to raise heavy loads.
- Force required to lift block is significantly less than block weight.
- Friction prevents wedge from sliding out.
- Want to find minimum force $P$ to raise block.

Block as free-body

\[ \sum F_x = 0 : \]
\[ - N_1 + \mu_s N_2 = 0 \]
\[ \sum F_y = 0 : \]
\[ -W - \mu_s N_1 + N_2 = 0 \]

or

\[ \sum \bar{R}_1 + \bar{R}_2 + \bar{W} = 0 \]

Wedge as free-body

\[ \sum F_x = 0 : \]
\[ - \mu_s N_2 - N_3(\mu_s \cos 6^\circ - \sin 6^\circ) + P = 0 \]
\[ \sum F_y = 0 : \]
\[ -N_2 + N_3(\cos 6^\circ - \mu_s \sin 6^\circ) = 0 \]

or

\[ \bar{P} - \bar{R}_2 + \bar{R}_3 = 0 \]
• Point of wheel in contact with ground has no relative motion with respect to ground. Ideally, no friction.

• Moment $M$ due to frictional resistance of axle bearing requires couple produced by equal and opposite $P$ and $F$.

Without friction at rim, wheel would slide.

• Deformations of wheel and ground cause resultant of ground reaction to be applied at $B$. $P$ is required to balance moment of $W$ about $B$.

$$Pr = Wb$$

$b = \text{coef of rolling resistance}$
A pulley of diameter 4 in. can rotate about a fixed shaft of diameter 2 in. The coefficient of static friction between the pulley and shaft is 0.20.

Determine:

• the smallest vertical force $P$ required to start raising a 500 lb load,

• the smallest vertical force $P$ required to hold the load, and

• the smallest horizontal force $P$ required to start raising the same load.

**SOLUTION:**

• With the load on the left and force $P$ on the right, impending motion is clockwise to raise load. Sum moments about displaced contact point $B$ to find $P$.

• Impending motion is counterclockwise as load is held stationary with smallest force $P$. Sum moments about $C$ to find $P$.

• With the load on the left and force $P$ acting horizontally to the right, impending motion is clockwise to raise load. Utilize a force triangle to find $P$. 
Sample Problem 8.6

SOLUTION:

• With the load on the left and force $P$ on the right, impending motion is clockwise to raise load. Sum moments about displaced contact point $B$ to find $P$.

The perpendicular distance from center $O$ of pulley to line of action of $R$ is

$$r_f = r \sin \phi_s \approx r \mu_s \quad r_f \approx (1 \text{ in.})0.20 = 0.20 \text{ in.}$$

Summing moments about $B$,

$$\sum M_B = 0: \quad (2.20 \text{ in.})(500 \text{ lb}) - (1.80 \text{ in.})P = 0$$

$$P = 611 \text{ lb}$$
Sample Problem 8.6

- Impending motion is counter-clockwise as load is held stationary with smallest force $P$. Sum moments about $C$ to find $P$.

The perpendicular distance from center $O$ of pulley to line of action of $R$ is again 0.20 in. Summing moments about $C$,

$$
\sum M_C = 0 : \quad (1.80 \text{ in.})(500 \text{ lb}) - (2.20 \text{ in.})P = 0
$$

$$
P = 409 \text{ lb}
$$
Sample Problem 8.6

- With the load on the left and force $P$ acting horizontally to the right, impending motion is clockwise to raise load. Utilize a force triangle to find $P$.

Since $W$, $P$, and $R$ are not parallel, they must be concurrent. Line of action of $R$ must pass through intersection of $W$ and $P$ and be tangent to circle of friction which has radius $r_f = 0.20$ in.

\[
\sin \theta = \frac{OE}{OD} = \frac{0.20 \text{ in.}}{(2 \text{ in.}) \sqrt{2}} = 0.0707
\]

\[
\theta = 4.1^\circ
\]

From the force triangle,

\[
P = W \cot(45^\circ - \theta) = (500 \text{ lb}) \cot 40.9^\circ
\]

$P = 577 \text{ lb}$
Belt Friction

- Relate $T_1$ and $T_2$ when belt is about to slide to right.
- Draw free-body diagram for element of belt

\[
\sum F_x = 0: \quad (T + \Delta T) \cos \frac{\Delta \theta}{2} - T \cos \frac{\Delta \theta}{2} - \mu_s \Delta N = 0
\]

\[
\sum F_y = 0: \quad \Delta N - (T + \Delta T) \sin \frac{\Delta \theta}{2} - T \sin \frac{\Delta \theta}{2} = 0
\]

- Combine to eliminate $\Delta N$, divide through by $\Delta \theta$,

\[
\frac{\Delta T}{\Delta \theta} \cos \frac{\Delta \theta}{2} - \mu_s \left( T + \frac{\Delta T}{2} \right) \sin \left( \frac{\Delta \theta}{2} \right) \frac{\sin \left( \frac{\Delta \theta}{2} \right)}{\Delta \theta/2}
\]

- In the limit as $\Delta \theta$ goes to zero,

\[
\frac{dT}{d\theta} - \mu_s T = 0
\]

- Separate variables and integrate from $\theta = 0$ to $\theta = \beta$

\[
\ln \frac{T_2}{T_1} = \mu_s \beta \quad \text{or} \quad \frac{T_2}{T_1} = e^{\mu_s \beta}
\]
A flat belt connects pulley $A$ to pulley $B$. The coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$ between both pulleys and the belt.

Knowing that the maximum allowable tension in the belt is 600 lb, determine the largest torque which can be exerted by the belt on pulley $A$.

**SOLUTION:**

- Since angle of contact is smaller, slippage will occur on pulley $B$ first. Determine belt tensions based on pulley $B$.
- Taking pulley $A$ as a free-body, sum moments about pulley center to determine torque.
Sample Problem 8.8

SOLUTION:

- Since angle of contact is smaller, slippage will occur on pulley \( B \) first. Determine belt tensions based on pulley \( B \).

\[
\frac{T_2}{T_1} = e^{\mu_s \beta} \quad \frac{600\text{lb}}{T_1} = e^{0.25(2\pi/3)} = 1.688
\]

\[
T_1 = \frac{600\text{lb}}{1.688} = 355.4\text{lb}
\]

- Taking pulley \( A \) as free-body, sum moments about pulley center to determine torque.

\[
\sum M_A = 0: \quad M_A + (8\text{in.})(355.4\text{lb} - 600\text{lb}) = 0
\]

\[M_A = 163.1\text{lb} \cdot \text{ft}\]